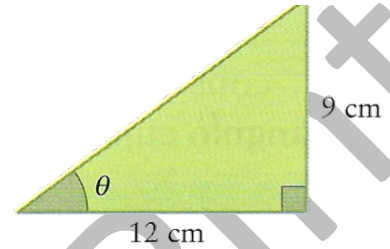
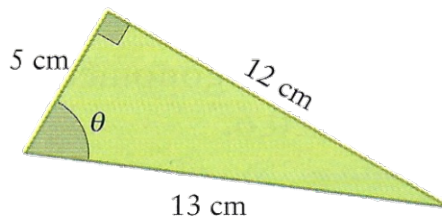
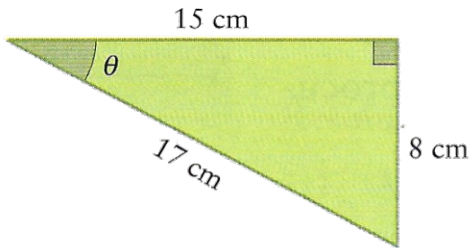


Matemática 12º ano

Revisões de trigonometria 11º ano ----- Prof. Mónica Pinto

1. Para cada um dos triângulos retângulos escreva as razões trigonométricas do ângulo θ .

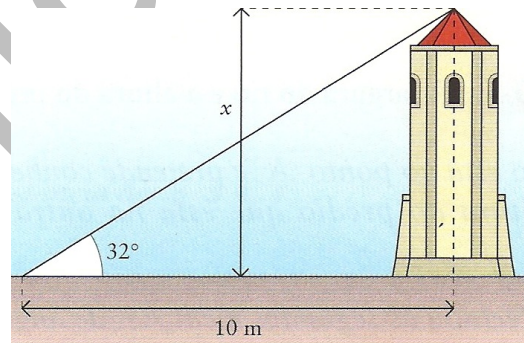
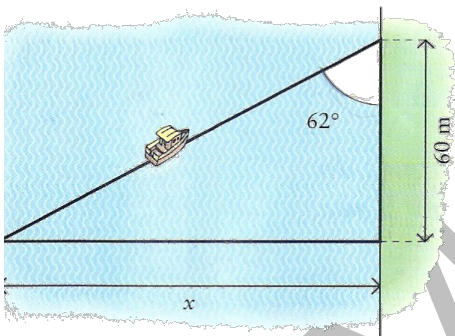


Sol. a. $\sin \theta = \frac{8}{17}$ $\cos \theta = \frac{15}{17}$ $\tan \theta = \frac{8}{15}$ b. $\sin \theta = \frac{12}{13}$ $\cos \theta = \frac{5}{13}$ $\tan \theta = \frac{12}{5}$ c. $\sin \theta = \frac{3}{5}$ $\cos \theta = \frac{4}{5}$ $\tan \theta = \frac{3}{4}$

2. Observa as figuras e, de acordo com os dados, determina:

a. A largura do rio

b. a altura da torre



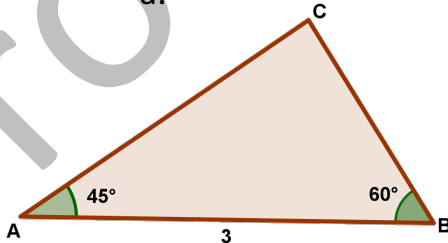
Soluções:

a. 112,84m , b. 6,25m

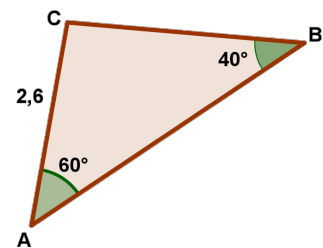
3. Tendo em conta os dados da figura em que a medida do comprimento dos lados está expressa numa dada unidade, resolve cada um dos seguintes triângulos [ABC].

Apresenta, quando necessário, valores aproximados à décima do grau, para a amplitude dos ângulos, e à décima da unidade, para o comprimento dos lados.

a.



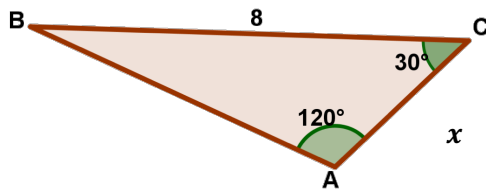
b.



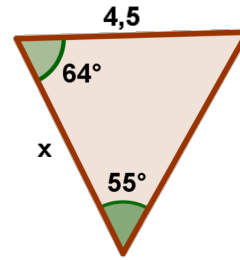
Sol. a. $\hat{A}CB = 75^\circ$, $\overline{AC} \approx 2,7$, $\overline{BC} \approx 2,2$ b. $\hat{A}CB = 80^\circ$, $\overline{AB} \approx 4,0$, $\overline{BC} \approx 3,5$

4. Determina x , tendo em conta as figuras apresentadas:

a.



b.



Sol. a. $\frac{8\sqrt{3}}{3}$ b. $\approx 4,8$

5. Seja α um ângulo agudo tal que $\sin \alpha = \frac{2}{3}$.

Calcula:

a. $\cos \alpha$.

b. $\operatorname{tg}^2 \alpha + 1$

Sol a. $\frac{\sqrt{5}}{3}$ b. $\frac{9}{5}$

6. Calcula, sem recorrer à calculadora:

a. $2 \sin \pi - \frac{1}{3} \sin\left(\frac{5}{2}\pi\right) - 4 \sin\left(\frac{3\pi}{2}\right) + \cos(2\pi)$

d. $-\sin\left(\frac{8\pi}{3}\right) + \cos\left(\frac{-10\pi}{3}\right) + 2 \sin\left(\frac{-9\pi}{4}\right) - \operatorname{tg}\left(\frac{3\pi}{4}\right)$

b. $\cos\frac{\pi}{4} + 5 \sin(2\pi) - \cos(2001\pi)$

e. $\sin 450^\circ + \cos 720^\circ$

c. $5 \operatorname{tg}(3\pi) - 2 \cos\frac{\pi}{6} + 3 \sin\left(\frac{17\pi}{6}\right)$

f. $\operatorname{tg} 540^\circ - \sin 630^\circ$

Sol. a. $\frac{14}{3}$ b. $\frac{\sqrt{2}}{2} + 1$ c. $-\sqrt{3} + \frac{3}{2}$ d. $\frac{1}{2} - \frac{\sqrt{3}}{2} - \sqrt{2}$ e. 2 f. 1

7. Simplifica as expressões:

a. $\cos(\pi + \alpha) - \sin\left(\frac{\pi}{2} - \alpha\right) + \cos(3\pi - \alpha)$

Soluções: $-3 \cos \alpha$

b. $\cos(-5\pi + \alpha) + \operatorname{tg}(9\pi - \alpha) - \sin\left(\frac{7\pi}{2} - \alpha\right)$

$- \operatorname{tg} \alpha$

c. $\cos(3\pi + \alpha) - \sin\left(\frac{5\pi}{2} + \alpha\right) + \sin(-\alpha - \pi) + \cos\left(\frac{11\pi}{2} + \alpha\right)$

$-2 \cos \alpha + 2 \sin \alpha$

d. $\sin\left(\frac{3\pi}{2} - \alpha\right) + \operatorname{tg}(5\pi - \alpha) - 3 \cos(-\alpha) - \sin(7\pi + \alpha)$

$-4 \cos \alpha - \operatorname{tg} \alpha + \sin \alpha$

e. $\cos\left(\frac{\pi}{2} + \alpha\right) - \cos(\pi - \alpha) + \operatorname{sen}(5\pi - \alpha) + 4 \sin\left(\frac{9\pi}{2} + \alpha\right)$

$5 \cos \alpha$

f. $\sin(-\alpha - 3\pi) - \frac{1}{2} \cos(-\alpha) + 2 \sin(\pi + \alpha) - \cos(9\pi + \alpha)$

$-\sin \alpha + \frac{1}{2} \cos \alpha$

g. $\cos\left(\frac{3\pi}{2} + \alpha\right) \times \sin(-\alpha) + \sin^2(\pi + \alpha) - \operatorname{tg}(\alpha - \pi)$

$- \operatorname{tg} \alpha$

8. Sabendo que $\sin \alpha = -\frac{1}{3}$, $\alpha \in \left] \frac{\pi}{2}; \frac{3\pi}{2} \right[$,

determina o valor exato de $\sin\left(-\frac{\pi}{2} - \alpha\right) + \operatorname{tg}(\alpha + 4\pi)$

Sol: $\frac{11\sqrt{2}}{12}$

9. Sabendo que $\cos \alpha = -\frac{2}{3}$, $\alpha \in \left] -\pi; \frac{\pi}{2} \right[$,

determina o valor exato de $\cos\left(\frac{3\pi}{2} - \alpha\right) + \cos(-\pi - \alpha)$

Sol: $\frac{2+\sqrt{5}}{3}$

10. Resolva em \mathbb{R} e em radianos, as seguintes equações trigonométricas:

- | | | |
|-------------------------------------|---|---|
| a) $\cos(x) = \frac{\sqrt{3}}{2}$ | h) $\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) = 1$ | m) $\sin(2x) = \sin(3x)$ |
| b) $\cos(x) = 1$ | i) $\operatorname{sen}^2(x) = \frac{1}{2}$ | n) $\cos(2x) = \cos(3x)$ |
| c) $\operatorname{sen}(x) = 0$ | j) $\operatorname{tg}^2(x) = 3$ | o) $\sin^2(x) + \sin x = 0$ |
| d) $\operatorname{sen}(x) = 1$ | k) $\operatorname{sen}(5x) = \operatorname{sen}(2x)$ | p) $\cos(2x) + \operatorname{sen}(x) = 0$ |
| e) $5\operatorname{tg}(3x) - 5 = 0$ | l) $\cos\left(3x + \frac{\pi}{3}\right) = \cos(2x - 1)$ | q) $\operatorname{sen}^2(x) = 1 - 2\cos(x)$ |
| f) $\cos^2(x) = 1$ | | r) $\cos^2(4x) + 2\cos(4x) = -1$ |
| g) $8\sin x + 4 = 0$ | | |

Sol. a. $\left\{\frac{\pi}{6} + 2k\pi; -\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\}$ b. $\{2k\pi, k \in \mathbb{Z}\}$ c. $\{k\pi, k \in \mathbb{Z}\}$ d. $\left\{\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}\right\}$ e. $\left\{\frac{\pi}{12} + \frac{k\pi}{3}, k \in \mathbb{Z}\right\}$
 f. $\{k\pi, k \in \mathbb{Z}\}$ g. $\left\{-\frac{\pi}{6} + 2k\pi; \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\}$ h. $\left\{-\frac{\pi}{2} + 2k\pi; 2k\pi, k \in \mathbb{Z}\right\}$ i. $\left\{\frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$
 j. $\left\{-\frac{\pi}{3} + k\pi; \frac{\pi}{3} + k\pi, k \in \mathbb{Z}\right\}$ k. $\left\{\frac{2k\pi}{3}; \frac{\pi}{7} + \frac{2k\pi}{7}, k \in \mathbb{Z}\right\}$ l. $\left\{1 - \frac{\pi}{3} + 2k\pi; -\frac{1}{5} - \frac{\pi}{15} + \frac{2k\pi}{5}, k \in \mathbb{Z}\right\}$
 m. $\left\{-2k\pi; \frac{\pi}{5} + \frac{2k\pi}{5}, k \in \mathbb{Z}\right\}$ n. $\left\{-2k\pi; \frac{2k\pi}{5}, k \in \mathbb{Z}\right\}$ o. $\left\{-\frac{\pi}{2} + 2k\pi; k\pi, k \in \mathbb{Z}\right\}$ p. $\left\{\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + \frac{2k\pi}{3}, k \in \mathbb{Z}\right\}$
 q. $\left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$ r. $\left\{\frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}\right\}$

11. Resolva as seguintes equações nos conjuntos indicados:

- | | |
|--|---|
| a. $1 + 2\cos(2x) = 0$, em $[-\pi, 2\pi]$ | $S = \left\{-\frac{2}{3}\pi; -\frac{\pi}{3}; \frac{\pi}{3}; \frac{2}{3}\pi; \frac{4}{3}\pi; \frac{5}{3}\pi\right\}$ |
| b. $\sin(\pi x) + \cos(\pi x) = 0$, em $] -1, 1[$ | $S = \left\{-\frac{1}{4}; \frac{3}{4}\right\}$ |
| c. $3\operatorname{tg}\left(x + \frac{\pi}{3}\right) + \sqrt{3} = 0$, em $] -\pi, \pi[$ | $S = \left\{-\frac{\pi}{2}; \frac{\pi}{2}\right\}$ |

12. Determina:

- | | | |
|--|---|-----------------------------|
| a) $\arcsin\left(-\frac{1}{2}\right)$ | e) $\arccos\left(\frac{\sqrt{2}}{2}\right)$ | i) $\sin(\arctan(1))$ |
| b) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ | f) $\arccos(-1)$ | j) $\cos(\arctan \sqrt{3})$ |
| c) $\arcsin(-1)$ | g) $\sin\left(\arccos\left(\frac{1}{2}\right)\right)$ | |
| d) $\arcsin\left(\sin \frac{5\pi}{3}\right)$ | h) $\tan(\arctan(-5))$ | |

Sol. a. $-\frac{\pi}{6}$ b. $\frac{\pi}{3}$ c. $-\frac{\pi}{2}$ d. $-\frac{\pi}{3}$ e. $\frac{\pi}{4}$ f. π g. $\frac{\sqrt{3}}{2}$ h. -1 i. $\frac{\sqrt{2}}{2}$ j. $\frac{1}{2}$

13. Escreve em função de x ou escreve o valor exato:

a) $\cos(\arcsin(2x))$

b) $\sin(\arccos x)$

c) $\tan(\arccos x)$

d) $\cos(\arcsin(-\frac{3}{5}))$

e) $\tan(\arccos(\frac{2}{3}))$

Sol. a. $\sqrt{1-4x^2}$ b. $\sqrt{1-x^2}$ c. $\pm\sqrt{\frac{1-x^2}{x^2}}$ d. $\frac{4}{5}$ e. $\frac{\sqrt{5}}{2}$

14. Determina o período das seguintes funções trigonométricas:

a) $1 + 2 \sin\left(2x + \frac{\pi}{4}\right)$

b) $-2 \cos\left(\frac{x}{2}\right)$

c) $\tan\left(3x - \frac{\pi}{5}\right) + 2$

Sol. a. π b. 4π c. $\frac{\pi}{3}$